problem, which is restricted to a finite time interval, is repeated at a high rate, high enough to be presented on a normal electronic oscilloscope. This can be done by using a proper time scale. Of course, special components have to be designed for this purpose to meet the extremely high dynamic requirements. Computers of this type are primarily used to find qualitative quick problem solutions. They are also advantageous to find quickly a large sample of solutions in statistical investigations. However, the high precision, real-time, \textit{DC} computer is considered representative of present day analog computation. All further discussion will therefore concentrate on this specific form of Analog Computer.

3.24. Components of the DC Analog Computer

3.241. The High Gain Amplifier. It has already been pointed out that the high gain amplifier is the most important building block in \textit{DC} analog computation. By its very action it serves to make all computing processes more accurate and independent from each other. But to be able to do this the high gain amplifier must be able to meet a set of very stringent requirements. They are the following:

a) Extremely high gain. Gain values up to the order of $10^8$ are common in modern amplifiers.

b) Minimization of Drift. Drift is a complex combination of influences from different sources, but it is common practice to refer to it as being originated at the grid point of the first tube. It is possible today to obtain drift values of 100 microvolts, or smaller, in one 8-hour period, in an amplifier configuration with an effective gain of 1. In an integrator, the drift can be kept smaller than 0.1 V per hour.

c) A wide pass band extending from zero frequency to sufficiently high frequencies. The high frequency limit depends on the way of operation. It is usual to define it for feedback conditions which would reduce the effective gain to 1. Under such conditions the frequency response of modern amplifiers is flat within 1 dB and does not exceed a phase shift of 1° up to more than 10 KC.

d) Stability under all foreseeable feedback and loading conditions. This requirement is fundamental but is difficult to meet. The means to provide stability are proper compensation networks, the tolerances of which must be carefully considered.

e) Minimization of noise. Great care must be taken to reduce the effective noise to its theoretical minimum. Values achieved in modern amplifiers are less than 10 microvolts rms, referred to the grid of the first tube.

f) Sufficient output power. Modern amplifiers are capable of delivering 20 mA at the peak value of the output voltage, namely, 100 volts.

There are two basic operational principles which can be used in the design of a \textit{DC} amplifier. One is the direct coupling principle. This design is sensitive to drift. Improvements are made by using bridge balance circuits and stabilized power supplies. The other is the auxiliary carrier principle in which the original \textit{DC} variable is modulated on a carrier, amplified, and demodulated. This principle is essentially drift free, since the drift components originating in the amplifier itself are outside the pass band of the system, but there are limitations with respect to the frequency range of the variables. Obviously, the frequency of the variable must be small with respect to the frequency of the auxiliary carrier and the carrier frequency must be selected in such a way that adequate precision modulators and demodulators can be made available. Modern amplifiers use a combination of both principles as shown in Fig. 13. Here (1) is the direct coupled part, (2) is the \textit{AC} amplifier part, (3) and (4) the modulator and demodulator, including the auxiliary carrier source, (5) a coupling stage, (6) are blocking
capacitors which prevent DC currents from passing in and out from the AC amplifier, and (7) is a filter to eliminate the ripples after demodulation. Normally (1) has a gain of \(10^6\) and (2) a gain of \(10^8\). So the total gain at DC and low frequencies is approximately \(10^8\) and at higher frequencies about \(10^6\). The sensitive parts in such an amplifier are the modulator and demodulator. The modulator must be free from spurious offset voltages and must be well shielded to avoid induction of carrier components which would yield unwanted demodulation products. The best components which are presently available are mechanical devices. They are either vibrating relays, so called choppers, or rotating mechanical switches which can be used in common for a group of amplifiers. Normally, carrier frequencies around 100 cycles are used in the chopper system and frequency lower than 10 cycles for the rotation switch system. Fig. 14 and 15 show examples of practical amplifiers.

It was already pointed out that these high gain amplifiers are operated in a feedback arrangement to perform the desired computation processes. Fig. 16 shows the basic arrangement. The triangular symbol represents the high gain amplifier with gain, \(a\), as it was described in the block diagram of Fig. 12. The \(\{0\}\) represent admittances which operate on the input and output voltages and produce the currents, \(i_1\) and \(i_f\). The outputs of the two operational networks are connected to the input point of the high gain amplifier or the "summing point". Due to the high gain, the voltage, \(e'\), which is required to produce the desired output voltage, \(e_0\), is very small. In approximative descriptions of the process it is normally assumed that \(e'\) is equal to zero, or, that the two currents, \(i_1\) and \(i_f\), which flow to the summing point, are equal. Based on such an approximation, equation (9) describes the input-output relation for the overall combination.

\[
\begin{align*}
e' &= 0 \\
i_1 + i_f &= 0 \\
e_1 \{0_1\} + e_0 \{0_f\} &= 0
\end{align*}
\]

\[
\frac{e_0}{e_1} = - \frac{\{0_1\}}{\{0_f\}}
\] (9)
It is seen that this input-output relation is defined only by the operational networks. The sacrifice in effective gain is traded for an increasing independence on the properties of the amplifier itself. This in turn explains why the input-output relation of such a feedback device is practically independent of the loading conditions. Despite the reduction of gain of the amplifier itself due to finite loads, the effective gain of the complete circuit is practically not affected.

In consideration of the feedback properties the discussion of the main linear computing components can concentrate on the properties of the operational admittances.
In the cases of sign changer, adder, and amplifier the admittances are of the form \( \frac{1}{R} \) (see Fig. 16, a and c). So, the overall gain is described by the equation:

\[
i_t + \Sigma i_i = 0
\]

\[
e_0 = - Rf \Sigma \frac{e_i}{R_i}
\]

(10)

For the sign changer the two resistors are of the same value, usually about one Megohm. In the case of the amplifier the two resistors are selected in such a way as to yield the desired gain factor.

In all these cases the accuracy is defined by the properties of the resistors. The techniques to build precision resistors at reasonable cost are quite advanced. They are of the wire wound type, have small capacitances and small inductances. Temperature influences are kept under control by making the temperature coefficients of the resistors as equal as possible. In addition, these resistors are normally mounted in a temperature controlled environment, the "ovens", where temperature is kept constant within small tolerances.

Aging effects are greatly reduced by subjecting the resistors to a number of temperature and load "cycles" before they are built in.

With all these precautions it is now possible to keep the long term error in resistance smaller than \( 10^{-4} \) of the nominal value.

In integrators, the feedback admittance is of the form \( \omega C \) (see Fig. 16d).

\[
C \frac{de_o}{dt} + \frac{e_i}{R_i} = 0;
\]

\[
e_0 = - \frac{1}{CRf} \int e_i dt + e_0 (0)
\]

(11)

To build highly precise capacitors is extremely difficult. However, by mounting the capacitors in a temperature oven and by subjecting them to temperature and load cycles before installation, the errors in capacity can be reduced to smaller than \( 10^{-4} \) of the nominal value. But there is an additional requirement for these capacitors, namely, an extremely high leakage resistance. In performing a computation process it is frequently required to "hold" the computation for a certain time in order to study and to read out the previous results. During such an interval the capacitor must maintain its charge within a very high degree of accuracy. Discharging influences origi-
inating in the connected circuitry are compensated by the feedback action. But the internal discharge of the capacitor must be kept within required tolerances.

3.242. **Attenuators.** Potentiometers are used to establish coefficients smaller than 1. They are normally of the multi-turn type and have high linearity and high resolution. However, they operate into finite loads and the loading reactions have to be considered in order to establish the coefficients with the required accuracy. To reduce such loading effects, the output of a potentiometer is usually connected to the input of an operational amplifier. But since the input impedance of an operational amplifier is finite (see Fig. 16), a high accuracy adjustment of a coefficient potentiometer must be performed under given load conditions. This is done by connecting the potentiometer and its load to a high precision bridge network into which the desired coefficient value is preset.

![Diagram](image)

Fig. 17. Time Division Multiplier. Basic Block Diagram and Waveforms

3.243. **Time Division Multiplier.** The time division multipliers belong to the class which was formerly identified as modulation multipliers. Specifically, they make use of pulse width and pulse amplitude modulation. The block diagram in Fig. 17 shows the basic principles in a simplified form. It is convenient to begin the explanation with the waveform, (c), which is a pulse train of an amplitude which is proportional to one of the input variables, \( x_2 \). It is width-modulated by a factor, \( \frac{T_1}{T_1 + T_2} \), which, in turn, is proportional to the other input variable,
impossible to insert such changes into the total body of commands in an automatic device. To sacrifice the operational flexibility of the patchboard in favor of a gain in setup-speed would be justified only in a very few special cases.

3.37. Planning of Computer Systems

It certainly became evident, that a wide variety of computing components and auxiliary devices is available. In order to obtain an installation which optimally meets the individual requirements, a careful systems planning is essential. Despite the fact that manufacturers offer defined types of computers, there is still sufficient leeway to tailor an installation to specific needs. Commercial computers cover a wide range with respect to size, capabilities and accuracy, most of them adopt a “building-block” feature and most manufacturers are willing and capable of providing desired modifications. Here an attempt will be made to outline the considerations which are important for planning an Analog Computer system.

The main feature of course is the character of the problems to be solved. But another item has to be considered first and is frequently overlooked. If

Fig. 27. Small Computer, Table Model (Heathkit)

taken into account properly it may lead to entirely different basic concepts of Analog Computer installations. It is the philosophy of approach to solving scientific and engineering problems which prevail in the agency planning to use the computers. Two main trends are possible in this respect: Organizational
units highly specialized in different techniques and organizational units grouped around projects (team work structure). In practical reality the organization will not be absolutely clear-cut, normally there will be some overlapping of the basic structural forms. So the answer to the questions raised in the following will be a

management decision. In the case of the primarily specialized organization it is logical to create a computation unit which with its special equipment and knowhow is available to solve the problems arising in any of the other units. In this case the computer installation will have to be centralized, large, flexible, highly automatized and manned with trained personnel in order to provide efficient operation. On the other hand, in an organization of prevalent team work structure, a decentralized arrangement of smaller computer units may be preferable which brings the computers as near as possible to the men who have to solve the problems. As was already pointed out, the successful operation of analog computers comes naturally to every engineer and physicist and it is always surprising what intimate understanding of a problem he can obtain if he does the computer work himself. This is significantly different from getting a formalistic answer to a specified question from people who quite naturally cannot have the profound understanding of the problem as the originating engineer himself. The decentralized approach is justified primarily in such cases where one team deals with one problem for a long time, proceeding from basic concepts to the detailed design. A computer assigned to such a team will be set up in a permanent fashion for the problem under study. It will be available to every team worker directly to answer his questions and so contribute immensely to the efficiency of the analytical work. The computer then can be considered as a special laboratory tool which is the basic philosophy of analog computation as was stated before. Of
course, there are practical limitations. If the problems assigned to a team are very large and complex, the economic feasibility of the decentralized solution may become questionable. Another factor may be the necessity of maintaining a high degree of computer utilization. Again the decentralized installation lags in this respect. But its imponderable advantages should be weighed carefully.
against the possible economic disadvantages. Industry offers a wide selection of small and medium sized machines, which are very adequate for the decentralized approach. Since most of them employ a building block principle, computer components which are not needed too frequently can be used by different teams on an exchange basis. This would permit the simulation of quite large problems in a flexible way at tolerable investment costs.

Centralized installations normally are large and comprehensive and have to be designed for maximum efficiency. This requires high flexibility in changing from one problem to the next, avoiding idle computer time. The machine must work reliably to minimize trouble shooting time. It will be advantageous to use all available automatic features for setting, checking, programming and reading out. A strict routine maintenance scheme will have to be set up. Use should be made of test benches and other checkout facilities which permit to test and to repair computer components without interfering with the operation of the computer itself. In general, maintenance requirements are very similar to those encountered in the operation of communication equipment. This pertains to procedures, number and qualifications of personnel.

The planning of a centralized facility must be based on a sound estimate of the size and character of the problems to be solved immediately and in the
future. The appraisal of the size of the expected problems is important to determine whether it will be feasible to sub-divide the installation into a number of "stations", which can be used together to solve a large problem, or, individ-

![Fig. 31. Large Computer Installation (Beckman/EASE Computers)](image)

ually to solve a number of smaller problems simultaneously. Such a station must then be capable of working as a complete, independent computer, and it must be possible to combine it with other stations for larger problems. This requires a sufficient number of connecting trunk lines and operational stability of computer components operating with such lines, which in turn implies a carefully planned grounding system. If the stations work together, it must be possible to exercise operational control over all stations from any of the other ones. In short, it will not be sufficient to merely install a conglomeration of computer components, but it is necessary to plan a system which meets the requirements optimally. Such a system quite certainly will have many features which were not mentioned but which are typical for the agency which is planning the installation and the problems which have to be solved. All of them have to be carefully considered. But the experience of computer manufacturers results in a wide variety of meticulously planned computer units which meet the requirements as building blocks for most practical computer systems.

In assessing the manpower requirements for a centralized computer installation it seems to be profitable to consider the establishment of an analytical section. It consists of engineers and scientists who are well familiar with the problems to be solved, with all details of the computers and with all pertinent analytical procedures. They are not computer operators and their task is to give advice to the users of the computers with respect to the best computational procedure and to discuss formulation and possible simplifications of the problems. They design special check procedures, analyze problem solutions and establish cross-check computations in order to clarify question areas. They have to be
familiar with error-analytical methods and have to apply them as extensively as possible. This group maintains the intelligent human control over the mechanistic processes performed by a large computer installation.

![Large Computer Installation](image1)

Fig. 32. Large Computer Installation (Electronic Associates, Inc.)

![REAC Computer Installation](image2)

Fig. 33. REAC Computer Installation (Reeves Instrument Corp.)

### 3.4. Operation of a DC Analog Computer

#### 3.4.1. Introduction

The discussion so far was concerned with providing a basic understanding of analog computing techniques, of the equipment involved, and of its organization. The remaining important area to be covered now is the fruitful use of an analog computer. It is not enough to explain the operation of a computer by