Feedback

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Operations which form, in the causal sense, a closed sequence or chain, and which thus occur in a completed ring or loop, constitute a special branch of dynamics of great importance in controlling and regulatory devices. Such causal loops may be entirely automatic, or may contain one or more human elements as an essential connecting link. The term feedback has come to be applied in the identification of systems which are so constructed; a servomechanism is basically of this character.

The most important property of such a physical arrangement is the stability with which it operates. In cases where the components and the interconnections involved are linear, that is to say where the principles of additivity or superposition for cause and effect apply, the criteria and the instrumental stratagems for stability are well understood. The power of feedback processes, thus restricted, is considerable. They are applied for a variety of computational purposes, as where a characteristic is to be reciprocated or inverted, and for example, in certain complicated problems of smoothing and prediction.

On the other hand when nonlinear components are involved, and where consequently the additivity principle is violated, as in general with a human operator forming a connecting link in the loop, a considerably more recondite situation is attained. Beyond the inferences which are available from linear approximations, and which must be very carefully drawn, the major recourse for such questions has been to experimental methods and to model studies.

For purposes of broad reference, and to set down concretely a rather purified example of the feedback phenomenon, we give here briefly what may be considered the simplest and most fundamental system in which feedback is involved (Fig. 1).

The operator Φ, being the characteristic in the *box*, is of arbitrary nature and is merely assumed to connect dynamically the incoming and

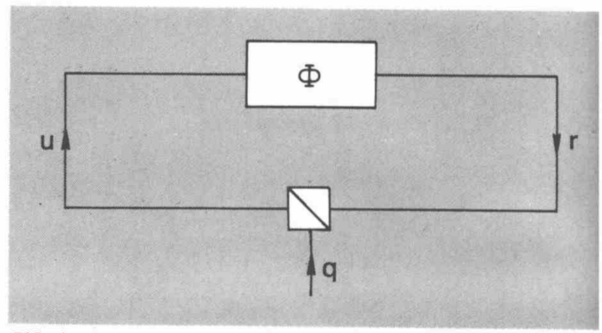

FIG. 1. Elementary Feedback loop

outgoing variables. Thus the operator Φ is a physically realizable functional which determines the local output as a function of time when the local input is predicated as a function of time. The input variable $q = q(t)$ is arbitrary. The output of the box is the variable r which is also the response to the operation Φ when performed on the input to the box. This is expressed mathematically by Eq. 1. Further, the variable u is the *error* or difference between q and r, as indicated by Eq. 2. These equations may be solved implicitly for u and r, as in Eqs 3 and 4, respectively.

$$r = \Phi \cdot u \qquad (1$$

$$u = q - r \qquad (2$$

$$u = q - \Phi \cdot u \qquad (3$$

$$r = \Phi \cdot q - \Phi \cdot r \qquad (4$$

The operator Φ may be linear, expressible for example, as a rational function of the derivative operator p. Then under appropriate restrictions u and r may be solved explicitly as in Eqs. 5 and 6.

$$u = \frac{1}{1+\Phi(p)} \cdot q \qquad (5$$

$$r = \frac{\Phi(p)}{1+\Phi(p)} \cdot q \qquad (6$$

Eqs. 5 and 6 give the familiar operators of linear feedback and servo theory. The roots o

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the rationalized denominators of these operators are significant to stability, while the merits of *performance* are obtainable in terms of the results of the entire operation when $q(t)$ is specified. Stability may also be studied, still under the linear restriction, through the exploration of $\Phi(p)$ itself, either experimentally or analytically. If linearity is not clearly indicated, Eqs. 3 and 4 are to be preferred.

A meaningful interpretation of the indicated basic closed causal system may be given as follows. Suppose it is required to duplicate q, which represents, for example, a variable weight in the pan of a balance. If then the response r is proposed as such a *duplicating* variable, the unbalance u indicates continually its failure in this regard. The operator Φ, which may now be considered a *follower*, adjusts r through interpretation of the error u. Thus the response r is a measure, better or worse, of the arbitrarily variable weight q. In more general cases the follower may have to operate through an additional chain of components, and q may contain irrelevant signals, and so on. All cases of feedback, by proper identification of the input quantity q and of the following operator Φ, reduce to this simple example.

George A. Philbrick has been one of the most far-sighted engineers of our time. His work with operational amplifiers and other signal-processing functional devices during the 1940s and 1950s contains the foundations of much modern electronic simulation and control.

Like many others, Mr. Philbrick worked for the government during the Second World War. His report at the end of that period was embodied as Part I of Volume III, of the summary Technical Report of Division 7 of the National Defense Research Committee (NDRC), of the Office of Scientific Research and Development (OSRD). The report was issued in 1946 and has since been declassified. In a recent conversation with ICS editor Alan Krigman, Mr. Philbrick commented that the report contained discussions of feedback and modeling, which he considered to be the basis of his engineering philosophy.

INSTRUMENTS AND CONTROL SYSTEMS is pleased to be able to present excerpts from this previously unpublished work. **Feedback** appears here, and the **Philosophy of Models** will be presented next month.