of the square of a given quantity, for which any conventional function generator (photoelectric,[32] cam, etc.) is applicable (see Chap. 10). (On the mechanical differential analyzer, squaring is performed by means of an

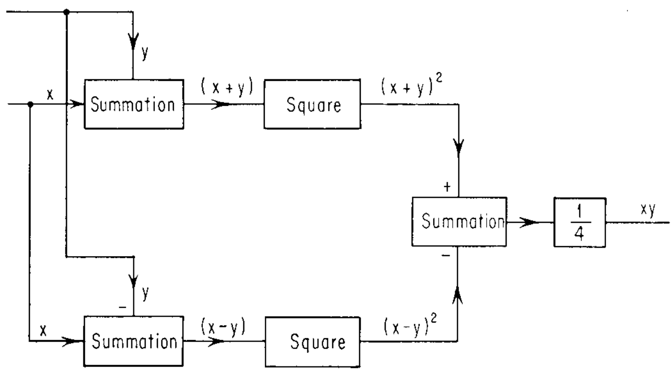

FIG. 5.33. Block diagram of square-law multiplier.

integrator in accordance with the relation: $x^2 = 2\int x\,dx$.) A block diagram of a square-law multiplier is shown in Fig. 5.33. We shall give three examples of this particular type.

Philbrick Multiplier. A multiplier employed in the Philbrick computer (Chap. 17) makes use of the square-law principle.[33,34] The squaring operation arises from the relation between the plate current i_p and the grid voltage e_g in a triode or pentode tube, assuming constant plate voltage (Fig. 5.34).[35] The characteristic

$$i_p = C_1 e_g + C_2 e_g^2 + C_3 e_g^3 + \cdots + C_n e_g^n \tag{5.130}$$

may be approximated in some tubes by a parabola over a prescribed range of grid voltage. The resulting voltage across the plate resistor is proportional to the square of the grid voltage. In the Philbrick computer, two 12AU7 triode sections are employed, one for positive and one for negative inputs. (The circuit is shown in Fig. 5.35.) The accuracy of this method is limited because of the deviation of the tube characteristic from a true parabola over the range of interest, which generally cannot be taken too large. One technique to extend accuracy is the use of two triodes in a

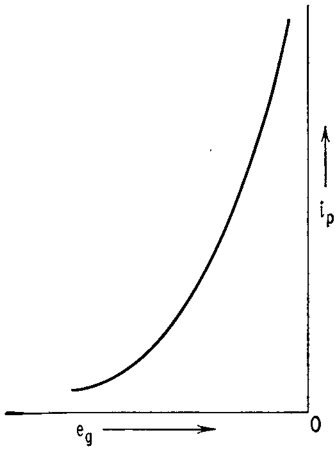

FIG. 5.34. Plate current-grid voltage characteristic.

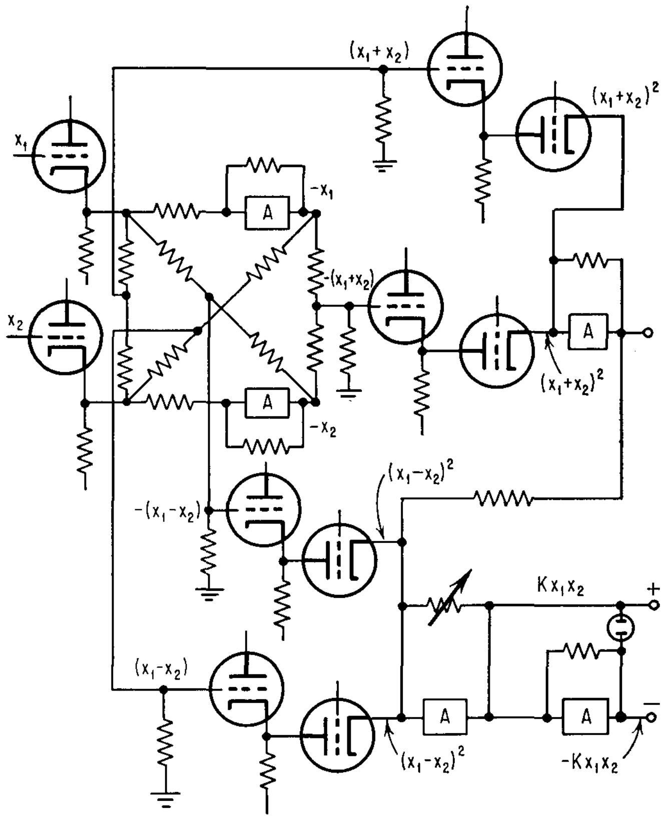

FIG. 5.35. Philbrick square-law multiplier.

balanced circuit in a way to subtract out odd powers in the series expansion for the plate current.

Thyrite. The squaring operation may be carried out by a nonlinear resistor called Thyrite.*[36] Thyrite, made of silicon carbide, has a volt-ampere characteristic of the form $i = ke^n$; the typical value for the exponent n ranges between 2.49 and 3.36.[37] To obtain the desired exponent $n = 2$, a linear resistor R of the proper value is placed in series with the Thyrite, as shown in Fig. 5.36*a*. The voltage output e_o is proportional to the square of the voltage input e_i.

Thyrite yields a voltage characteristic which is symmetric with respect to the origin rather than to the y axis, as shown in Fig. 5.36*a*. An absolute value circuit, as shown in Fig. 5.36*b*, may be employed to obtain a symmetric square (see Chap. 7). When e_i is positive, the amplifier output $-e_i$ is negative and the upper diode conducts. When e_i is negative,

* Thyrite is a registered trademark of the General Electric Company.

the upper diode is cut off and the lower one conducts. Thus, the voltage inserted across the diode is always negative, yielding the characteristic of Fig. 5.36*b*. A complete multiplier unit is shown in Fig. 5.36*c*, where R_1 and R_2 compensate for differences in the two squaring circuits and for losses in gain, respectively.

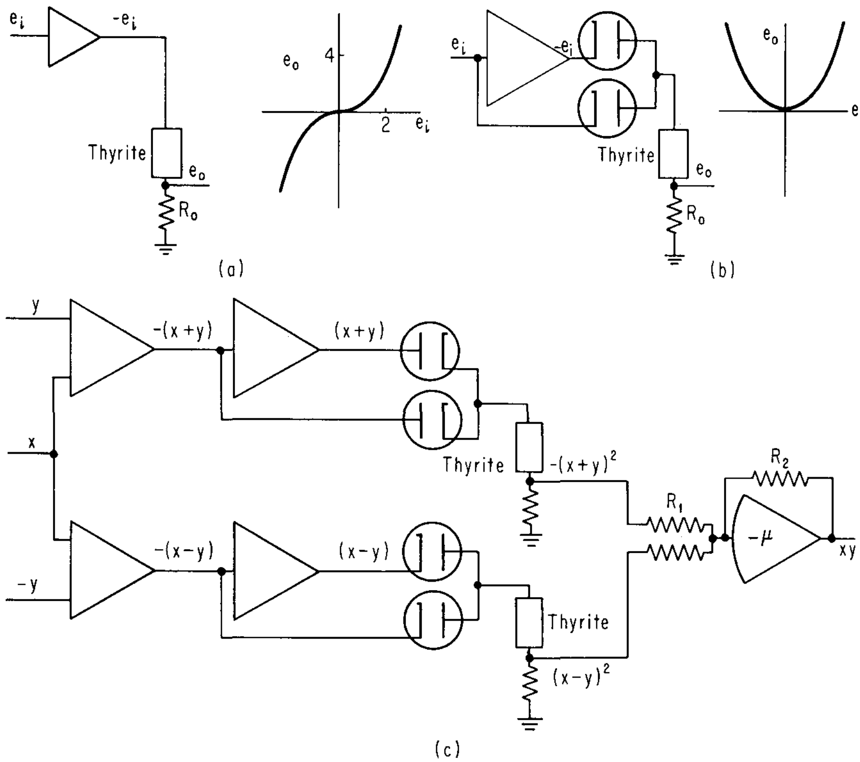

FIG. 5.36. Thyrite multiplier. (*a*) Voltage characteristic; (*b*) absolute value circuit; (*c*) multiplier circuit.

An accuracy of 1.25 per cent of full scale (10 volts) from zero frequency to a frequency of approximately 1,000 cps (with less than 1° of phase shift) may be realized.[25] Among the advantages of this multiplier are its high frequency response and its relatively low cost.

A square-law circuit involving a rectifier in series with a small resistance has also been employed.[38] It is based on the fact that the rectifier resistance R is related to the voltage (assumed low) across it by the expression

$$R = Ke^{-qV} + R_o, \tag{5.131}$$

where K, q, and R_o are constants.

Diodes. Diodes provide another means of squaring a given variable.[39–41] The parabola $y = x^2$ relating output to input is approximated by a polygonal type of curve, consisting of straight line segments. We shall employ here the method of Sec. 7.2, which is restricted to curves whose derivative is nonincreasing with increasing abscissa. To generate

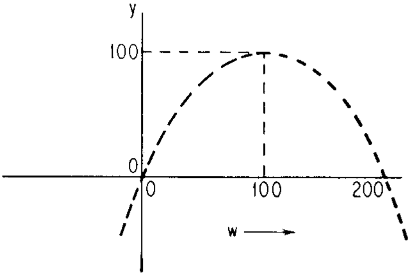

FIG. 5.37. Square-law characteristic.

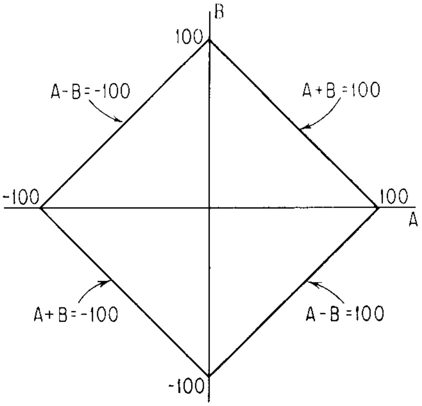

FIG. 5.38. Region of admissibility for A and B in the product AB.

a curve which satisfies this slope criterion, the independent variable x may be replaced by

$$w = 100 - x, \tag{5.132}$$

with

$$x \equiv x_1 = A + B \tag{5.133}$$

for one parabola, and

$$x \equiv x_2 = A - B \tag{5.134}$$

for the other. The equation for the parabola, as shown in Fig. 5.37, is

$$y(w) = 2w - \frac{w^2}{100} = w\left(2 - \frac{w}{100}\right), \tag{5.135}$$

and in terms of x

$$y(x) = (100 - x) \cdot \left[2 - \frac{(100 - x)}{100}\right] = 100 - \frac{x^2}{100}, \tag{5.136}$$

from which it is clear that

$$y(x_2) - y(x_1) = y(A - B) - y(A + B) = 100 - \frac{(A - B)^2}{100} - 100 + \frac{(A + B)^2}{100} = \frac{AB}{25}. \tag{5.137}$$

The curve $y(w)$ is symmetrical about the line $w = 100$, with intercepts at (0,0) and (200,0). The range of w lies between 0 and 200, i.e.,

$$0 \leq 100 - (A + B) \leq 200, \tag{5.138}$$
$$0 \leq 100 - (A - B) \leq 200. \tag{5.139}$$

The region of admissibility for A and B is given in Fig. 5.38.

A parabola can be fitted by means of this scheme in the range for which the slope of $y(w)$ does not become negative, i.e.,

$$0 \leq w \leq 100. \tag{5.140}$$

To double the range, define the function $\bar{w}$

$$\begin{aligned} \bar{w} &= w, & 0 \leq w \leq 100, \\ \bar{w} &= 200 - w, & 100 \leq w \leq 200. \end{aligned} \tag{5.141}$$

Now $y(\bar{w}) \equiv y(w)$. As w goes from 0 to 100, $\bar{w}$ covers the range 0 to 100; and as w goes from 100 to 200, $\bar{w}$ covers the same range in the reverse direction from 100 to 0. Thus, if we form $\bar{w}$ from w and fit a diode curve to $y(\bar{w})$ in the range 0 to 100, this is equivalent to fitting a curve to $y(w)$ in the range 0 to 200. Essentially, the diode arrangement is used twice, once forward and once backward. The transformation, as given in Eq. (5.141), may be carried out in many ways. One method employs a relay amplifier (Fig. 5.39) as described in Chap. 8. For $w < 100$, the relay is deenergized and $\bar{w}$ is equal to the voltage inserted in the NC contact, namely, w. For $w > 100$, the relay is energized and $\bar{w}$ is equal to the voltage inserted in the NO contact, namely, $200 - w$.

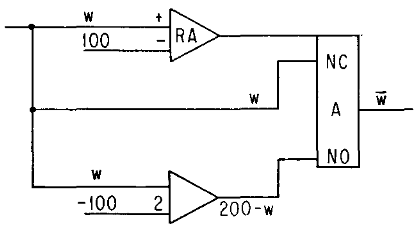

FIG. 5.39. Relay circuit.

The accuracy of this type of device is a function of the number of diodes used for the shaping. An accuracy figure of between 0.1 per cent and 1 per cent appears to be a realistic one. Critical operation of the unit occurs when one of the terms is zero, for in order to obtain zero output in this case perfect matching of the diode function generators is necessary. The diode multiplier does not possess the flexibility of devices, such as the servo or a-m f-m multiplier, which provide additional products of one variable by "ganging" potentiometers or slaves. In the present case, a complete unit must be used for each product. The chief virtue of the multiplier is its high frequency response; it is electronic and no filtering of the output is necessary.

5.9 LOGARITHMIC LAW

The logarithmic multiplier is similar conceptually to the square-law multiplier. The functions $\log x$ and $\log y$ are formed from x and y by means of diodes, function generators, etc. Summation of the two logs gives

$$\log x + \log y = \log xy, \tag{5.142}$$