STUDY OF SLOW SURFACE WAVE BY ANALOG COMPUTOR *

by S. *MUKHERJEE* **

SUMMARY. — The existence and nature of slow surface waves in a compressible plasma-dielectric interface is studied by analog computor. Use of the analog computor to solve such a boundary value problem, though unconventional, indicates the speed and usefulness of an analog computor to analyse the qualitative nature of a physical phenomena. The technique used is a « scanning » method in which multiple sweeping signals are used suitably to represent parameters of interest.

I. Introduction.

In recent years, we have become concerned about the generation of very high microwave frequencies due to electron beam through a plasma-filled wave guide, communication systems through extra-terrestrial space filled with ionized gas, communications to and from a moving vehicle (e.g., missiles and satellites) coated with a plasma layer formed particularly during its re-entry into the terrestrial atmosphere. These problems accentuated intensive study on plasma and its behavior in the boundaries. One of the studies of interest is the behavior of surface waves excited in plasma-dielectric interface. These waves (i.e., surface waves) have the propagating function along the interface. They do not radiate and are characterized by the decay away from the interface of the two media (fig. 1). In a research paper [1] the author reported a study of slow surface wave propagation along a compressible plasma slab. This article is the companion to the research report and describes the computational details where the use of an analog computor is quite new.

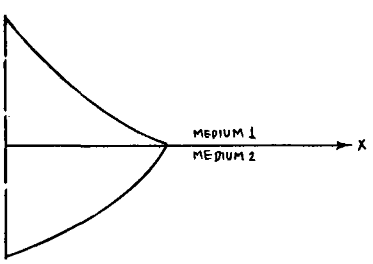

Fig. 1. — Surface Wave Phenomenon Defined.

The particular analog computor used for this project is the high-speed repetitive SK5-Computor. A description of the computor is given in a subsequent section. The characteristic feature of this computational procedure is that it preserved a direct relationship between the physical problem and the computor solution via their common mathematical analog — a true Man-Machine-Problem relationship.

The analog technique used is basically a « scanning method » — scanning a slow signal by an extremely fast signal. This method is favoured against using a host of potentiometers or decade boxes and function generators. Further details on the computation are given in Section IV, including the diagram of the computor set up to study surface waves. Unfortunately, in spite of the progress with the analog computor since its inception, no standard notations have been established or accepted by convention. Therefore, the following notations (fig. 2) which have been used during this project are suggested to the readers for reference and for understanding of the block diagrams. Signal flow is always from left to right.

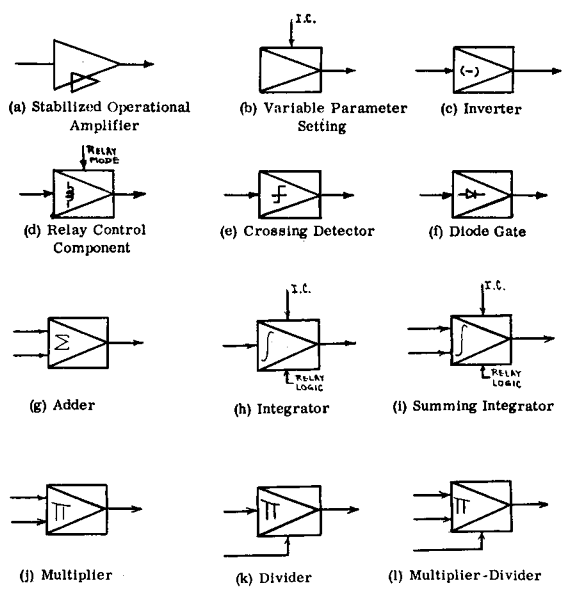

Fig. 2. — Operational Symbols.

Prior to giving further details on the computational technique, the physical problem is described in the next section. Based on the assumed physical model, a mathematical relation (known as « dispersion relation ») is obtained between the variables of interest. This relation is indeed the key to the study of slow surface wave phenomenon supposed to exist in the physical situation described. The analog computor is then set up to find the solution of the dispersion relation and the surface wave characteristics are displayed directly on the oscilloscope screen. Essentially, the object is to display the plot of the excitation radian frequency versus the wave number along the interface of the

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two media. From the slope of the curve, one can then immediately infer whether the slow surface wave is of the « forward » or « backward » type. The result is almost instantaneous.

II. Statement of the problem.

An isotopic collisionless compressible electron plasma slab of thickness « a » is considered as shown in figure 3. Medium 1 (i.e., plasma medium) is charac-

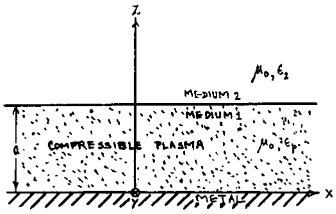

Fig. 3. — Geometry of the Problem.

terized by the permeability μ_0 and dielectric constant ε_p, while medium 2 by μ_0 and ε_2. The dielectric constant, ε_p, of the plasma is given by

$$\varepsilon_p = \varepsilon_0 \left[1 - \frac{\omega_p^2}{\omega^2}\right] \tag{1}$$

where

ε_0 = dielectric constant of the free-space = 8.854×10^{-12} farad/meter
μ_0 = permittivity of free-space = 1.257×10^{-6} henry/meter
ω_p = plasma-electron angular frequency (radian/sec)
$= e^2 \sqrt{n_0/m\,\varepsilon_0}$
e = magnitude of the electron charge, 1.602×10^{-19} coulomb
n_0 = average electron density
m = mass of an electron, 9.109×10^{-31} kilogram
ω = excitation frequency in radian/sec.

Details of mathematical formulation of the problem are purposely omitted here. It is shown [2] that the dispersion relation (or characteristic equation) obtained by satisfying the boundary condition at $z = 0$ and $z = a$ is given by

$$\frac{\nu_1 \dfrac{\omega^2 \nu_p}{\omega_p^2 \beta^2} \sin \nu_p a + \sin \nu_1 a}{\cos \nu_p a - \cos \nu_1 a} = \frac{\nu_1 \cos \nu_1 a - \nu_1 \cos \nu_p a - i\,\nu_2 \dfrac{\varepsilon_p}{\varepsilon_2} \sin \nu_1 a}{\nu_1 \sin \nu_1 a + \dfrac{\omega_p^2 \beta^2}{\omega^2 \nu_p} \sin \nu_p a + i\,\nu_2 \dfrac{\varepsilon_p}{\varepsilon_2} \cos \nu_1 a} \tag{2}$$

where

$$\left.\begin{aligned} \nu_1^2 &= \omega^2 \mu_0 \varepsilon_p - \beta^2 = k_1^2 - \beta^2 \\ \nu_2^2 &= \omega^2 \mu_0 \varepsilon_2 - \beta^2 = k_2^2 - \beta^2 \\ \nu_p^2 &= \frac{\omega^2 \varepsilon_p}{u^2 \varepsilon_0} - \beta^2 = k_p^2 - \beta^2 \end{aligned}\right\} \tag{3}$$

u = adiabatic sound velocity in the electron fluid (cms/sec)
ν_1 and ν_2 = wave numbers (in the z-direction) of the electromagnetic waves in media 1 and 2 respectively
ν_p = wave number (in the z-direction) of the plasma wave (also called acoustic wave) in medium 1
β = wave number in the x-direction which must be the same for both media
ε_2 = dielectric constant of medium 2 (values in range of 1 to 5 are considered for computation)

Now, in order to exhibit surface wave phenomena, at least one of the propagation wave numbers of the two media (plasma and dielectric, ε_2) must be imaginary. For this problem, the wave numbers ν_1 and ν_2 of the electromagnetic waves in media 1 and 2 respectively are considered to be imaginary, while the wave number ν_p of the plasma wave is considered real or imaginary as shown later in the three particular cases of study.

It is also well known that surface wave can exist when $\omega_p > \omega$ (i.e., ε_p is negative) and also when $\omega_p < \omega$ (i.e., ε_p is positive [3]). Both of these situations are considered in this problem. Based on these considerations, the following three cases are proposed for investigation.

Case 1. ε_p positive ; ν_1 and ν_2 imaginary; ν_p real.

Case 2. ε_p positive ; ν_1, ν_2 and ν_p imaginary. (4)

Case 3. ε_p negative; ν_1, ν_2 and ν_p imaginary.

III. Computor and Its Paraphernalia.

It is mentioned earlier that the analog computor involved in this study is the high-speed repetitive SK5-Computor. It consists of several basic classes of units, of which the following are utilized for this program:

(1) SK5-U Universal Linear Operator. It is the fundamental linear building block of the computor. It is used as an adder, integrator for bang-bang operation and variable parameter setting. Conceptually, the SK5-U can be represented by

$$e = 10^n \sum_{i=1}^{4} \pm a_i e_i \pm e_0 \tag{5}$$

as an adder

$$e = 10^m \int_0^t \pm a_i e_i \, dt \pm e_0 \tag{6}$$

as a summing integrator

where

e = output in volts
e_i = input signals in volts
a_i = input coefficients provided as decade switches, $0.00 \leqslant a_i \leqslant 10.00$ (nominal)
e_0 = initial condition in volts, $0.00 \leqslant e_0 \leqslant 100.0$ (nominal)
n = gain setting switch position, — 1, 0, 1 or 2. Provides gain of 0.1, 1, 10 and 100

m = time scale setting switch position, 0, 1, 2, 3 or 4. Provides time constants of 1, 0.1, 0.01, 0.001 and 0.0001 second.

(2) SK5-M Universal Multiplier/Divider. It is used as multiplier, divider and multiplier/divider combination. It represents its operation in the form

$$e = \frac{e_1}{100 + e_3} (10^n \, a \, e_2) \quad \text{as multiplier} \qquad (7)$$

$(100 + e_3) \leqslant +100$ volts, e_3 usually 0,

$$e = \frac{100 + e_1}{e_3} (10^n \, a \, e_2) \quad \text{as divider} \qquad (8)$$

$(100 + e_1) \leqslant +100$ volts, e_1 usually 0,

$e_3 > 0$ volts,

$$e = \frac{e_1}{e_3} (10^n \, a \, e_2) \quad \text{as multiplier/divider} \qquad (9)$$

where

$|e_1|, |e| \leqslant 100$ volts

$0 \leqslant a \leqslant 12.21$, coefficients provided in decade switches

$n = 0, -1$, provides scale factor multiplier, 1.0 and 0.1 respectively

$(10^n \, a \, e_2) \leqslant 100$ volts.

(3) SK5-H Operational Manifold. It consists of five uncommitted high-gain stabilized operational amplifiers. Each amplifier channel is independent. The input, output and other necessary terminals are brought in the front, as well as in the rear panel. For this problem, two amplifiers are used as a crossing detector and two for gate circuit.

(4) SK5-R Relay Control Component. It is used to energize the mechanical relays in SK5-U. There are two relays in each SK5-U to control its Mode (set, run or hold) when used as an integrator. A zero to twenty-volt square wave signal is required for the input.

(5) 5934 Display System. It enables the display of eight independent variables simultaneously on a 17" Cathode ray tube. The display period can be varied from 25 milliseconds to 50 seconds at eleven discreet steps. The output of the display provides a clamp signal to drive the relay control component (SK5-R) synchronously with the display sweeping frequency.

A Polaroid camera attached with the Display is used to take pictures (both positive and negative frames) of the results.

Programming of the computor is fairly simple. Since each unit is a mathematical operator, it is possible for the computor-man to detail the entire problem on a specified program sheet. A sample of these sheets used in this problem is exhibited in figure 4.

Fig. 4. — SK 5 Set-Up Sheet.

IV. Analog Computor and Investigation of Surface Wave.

It has been mentioned in Section II that there are three cases defined by (4) to be considered. Applying the necessary conditions in relation (2), the corresponding dispersion relations are obtained and the computor is set up accordingly.

Case 1. ε_p positive; ν_1 and ν_2 imaginary; ν_p real. Let $\nu_1 = i\,\eta_1$ and $\nu_2 = i\,\eta_2$, where η_1 and η_2 are real. Using these relations in (2), we get

$$\begin{aligned} 2\,\eta_1 - \Big[\Big(\eta_1^2 \frac{\omega^2 \nu_p}{\omega_p^2 \beta^2} - \frac{\omega_p^2 \beta^2}{\omega^2 \nu_p}\Big) \sinh \eta_1 a \\ + \Big(\eta_1 \frac{\omega^2 \nu_p}{\omega_p^2 \beta^2}\Big) \Big(\eta_2 \frac{\varepsilon_p}{\varepsilon_2}\Big) \cosh \eta_1 a\Big] \sin \nu_p a \\ = \Big[2\,\eta_1 \cosh \eta_1 a + \Big(\eta_2 \frac{\varepsilon_p}{\varepsilon_2}\Big) \sinh \eta_1 a\Big] \cos \nu_p a \end{aligned} \qquad (10)$$

Suppose the left- and right-hand sides of (10) are represented by F_1 and F_2 respectively. Then for each set of the values of each parameter, different sets of F_1 and F_2 can be obtained. However, the solutions of (10) are those values of the parameters for which $F_1 = F_2$. Accordingly, ω vs. β curve can be obtained for the condition $F_1 - F_2 = 0$. It is, however, convenient to make the scale of ω vs. β diagram dimensionless by studying ω/ω_p vs. βa curves. Thus, rewriting (10), we have the non-dimensional relation in the form:

$$\begin{aligned} 2\,x_1 - \Big[\Big(x_1^2 \frac{m^2 y}{z^2} - \frac{x^2}{m^2 y}\Big) \sin x_1 \\ + \Big(x_1 \frac{m^2 y}{z^2}\Big) \Big(x_2 \frac{n}{p}\Big) \cosh x_1\Big] \sin y \\ = \Big[2\,x_1 \cosh x_1 + \Big(\frac{x_2 n}{p}\Big) \sinh x_1\Big] \cos y \end{aligned} \qquad (11)$$

where

$$\left.\begin{array}{lll} x_1 = \eta_1 a & p = \varepsilon_2/\varepsilon_0 & x_2 = \eta_2 a \\ y = \nu_p a & & z = \beta a \\ & & m = \omega/\omega_p \\ & & n = \varepsilon_p/\varepsilon_0 \end{array}\right\} \qquad (12)$$

For computational purposes, $\eta_1 a$ and $\nu_p a$ are considered as the prime variable function. $\eta_1 a$ is represented by a fast ramp, while $\nu_p a$ by a slow ramp (fig. 5). In other words, when $\nu_p a$ is varying from its minimum to the assigned maximum value (10^4 radians $= 100$ V) in 10 second computor time, $\eta_1 a$ sweeps about 500 times from its minimum to its assigned maximum value. Thus, about 500 discreet values of $\eta_1 a$ and corresponding $\nu_p a$ values are obtained repetitively to find the solutions of (10).

The ramps for $\eta_1 a$ and $\nu_p a$, as well as $(\eta_1 a)^2$ and $(\nu_p a)^2$, are generated by SK5-U. For repetitive operation, the relays of SK5-U are driven by two separate SK5-R relay control components. The Clamp signals of these SK5-R's are also obtained separately. The SK5-R connected with the slow ramp is driven by the 5934 Display clamp output, while for the other SK5-R a clamp signal is generated by two SK5-U's (fig. 6).

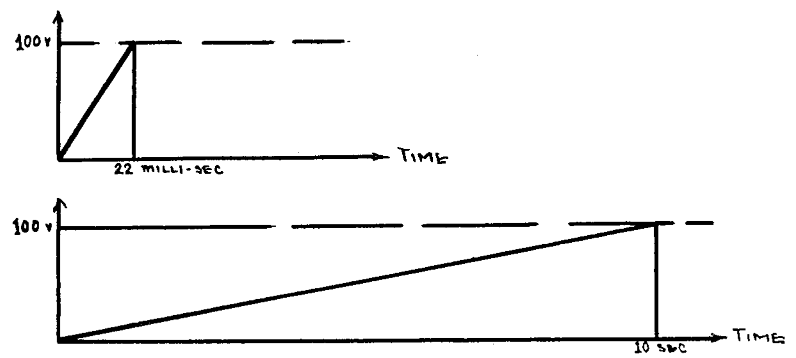

Fig. 5. — Ramp signals.

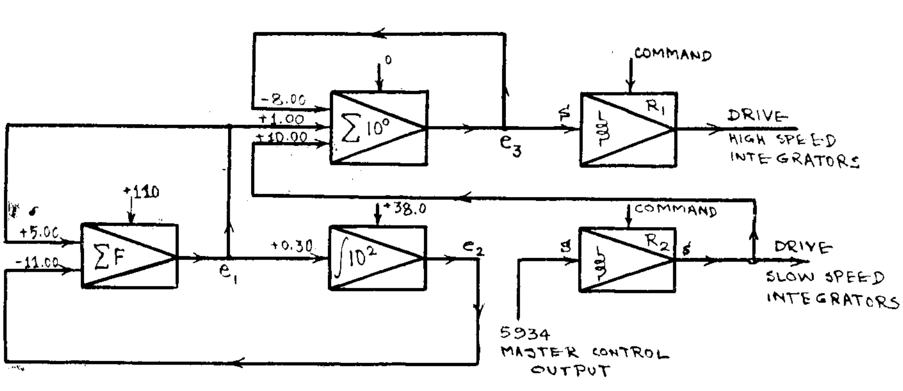

Fig. 6. — Astable Multivibrator.

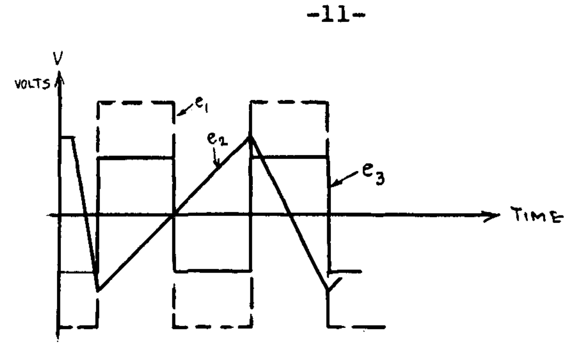

Fig. 7. — Wave Shapes for Figure 6.

Having $\eta_1 a$ and $\nu_p a$ as time dependent variables, it is quite easy to generate cosine, sine, cosh, sinh functions by SK5-U (fig. 8).

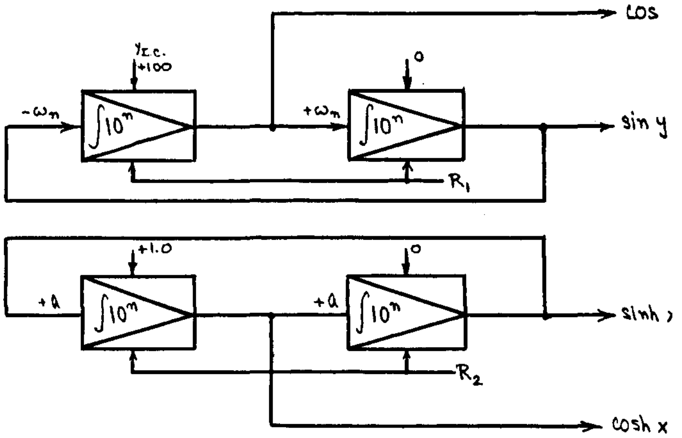

Fig. 8. — Generation of Sine, Cosine, Sinh and Cosh R_1 and R_2 refer to Figure 6.

ε_2 is considered as a parameter. A value of $1 \leqslant \varepsilon_2/\varepsilon_0 \leqslant 5$ is chosen and applied in discreet step simply by the decades provided in SK5-U. The values of $\eta_2 a$, βa, ω/ω_p and $\varepsilon_p/\varepsilon_0$ are derived from relation (3), choosing $c/u = 10^3$, $0.1 \geqslant (k, a)^2 \geqslant 0.01$ and their relationships with $\eta_1 a$, $\nu_p a$, $\varepsilon_2/\varepsilon_0$. Each of their values are thus possible to sweep repetitively for solutions of (11). Once all terms of the relation (11) are generated, the entire relation (11) can be realized by addition, multiplication and division.

Our purpose is to plot the ω/ω_p vs. βa curve when $F_1 - F_2 = 0$. To obtain these particular values, a crossing detector is designed (fig. 9) and driven by $(F_1 - F_2)$. The output of the crossing detector and its inverted signal are used as the gating signal for the Gate (Track & Hold Circuit - fig. 9). The signal representing ω/ω_p is fed into the Gate. Whenever the zero value occurs, i.e., $F_1 - F_2 = 0$, the Gate tracks this particular value of ω/ω_p and holds (for example, see figure 10). The output of the Gate can then be fed into a second Gate, the gating signal of which is in opposite polarity to that of the first. Thus, the second Gate *only* holds those values of ω/ω_p for which

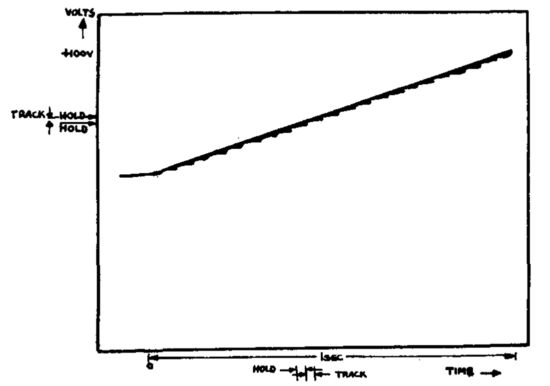

Fig. 10. — Ramp; track and hold output for $\nu_p a$ when $F_1 - F_2 = O$.

$F_1 - F_2 = 0$ (for example, see figure 11), and the output is fed to the 5934 Display. The values for βa are similarly obtained by another pair of Gates and the final output is fed into the display to obtain the required cross-plot. The complete computor diagram is

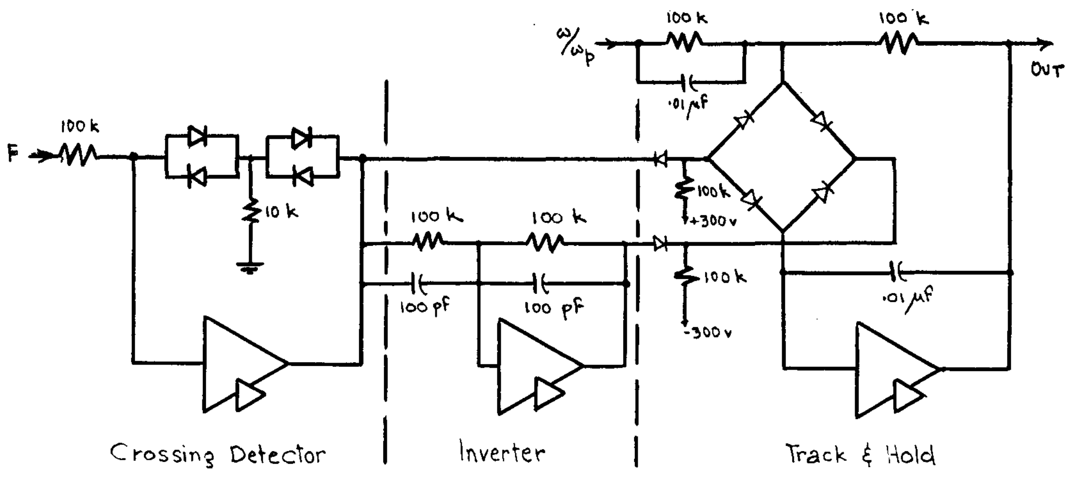

Fig. 9. — Crossing Detector, and Track and Hold Circuit.

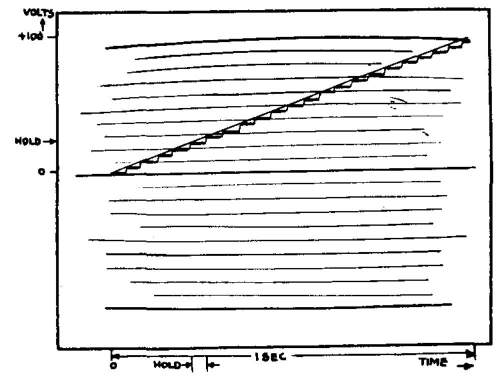

Fig. 11. — $v_p a$ ramp; hold output for $v_p a$ when $F_1 - F_2 = 0$.

illustrated in figure 12 and the ω/ω_p vs. βa curve in figure 13. The slope of the curve indicates that the surface wave is « forward » in nature. Figures 13 (a) and 13 (b) are representatives of a set of results obtained. In each illustration there are two curves. The bottom curve is for calibration purpose only. The curve shown on the top is of interest for this problem. Figure 13 (b) is the same as figure 13 (a) except that its horizontal scale is amplified four times for a better visual inspection. Further interpretations of these curves in relation to the problem are beyond the scope of this paper and, therefore, are omitted.

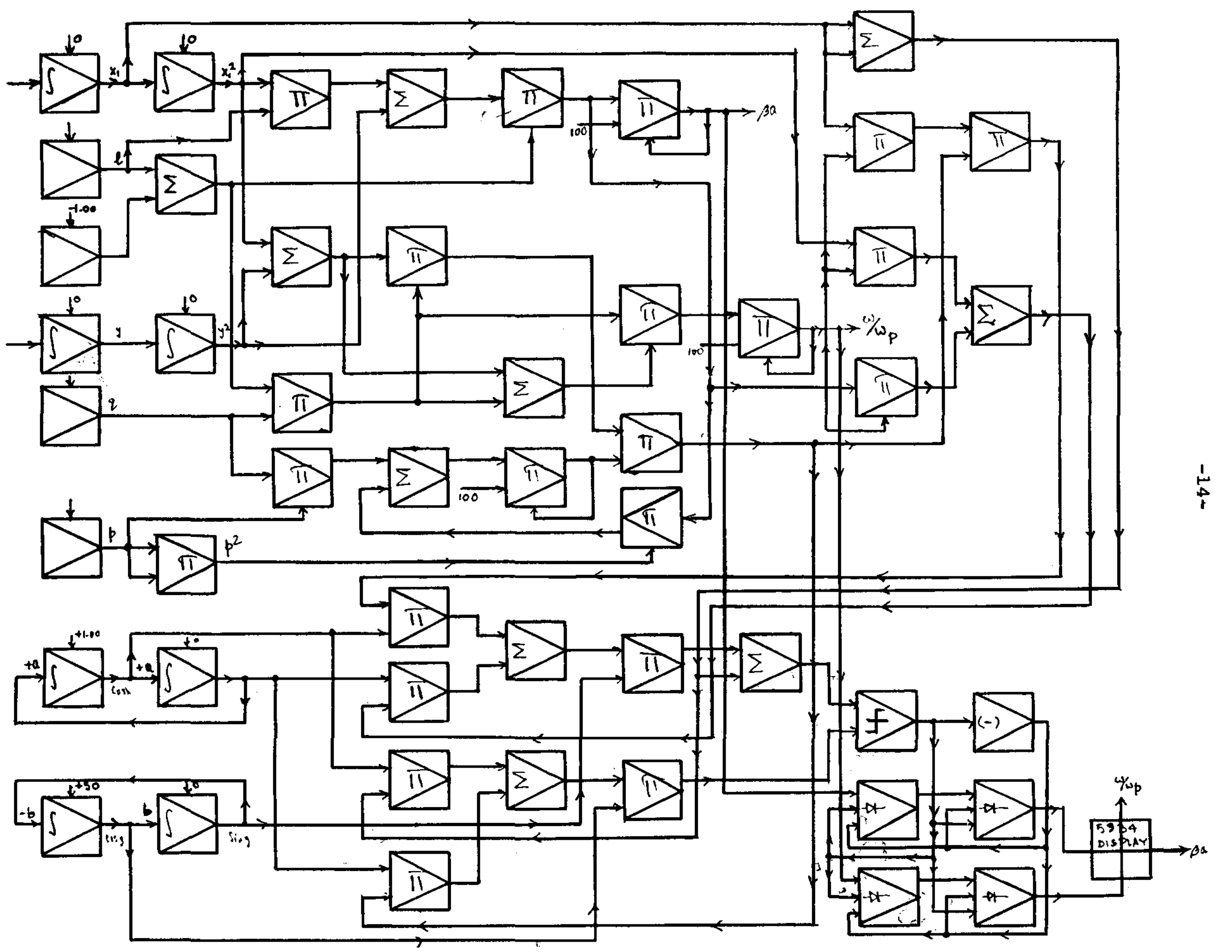

Fig. 12. — Computor Set-Up for Surface Wave Study.

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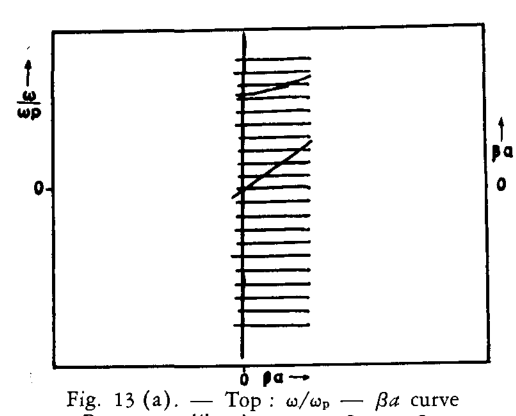

Fig. 13 (a). — Top : ω/ω_p — βa curve
Bottom : calibration curve βa vs. βa.

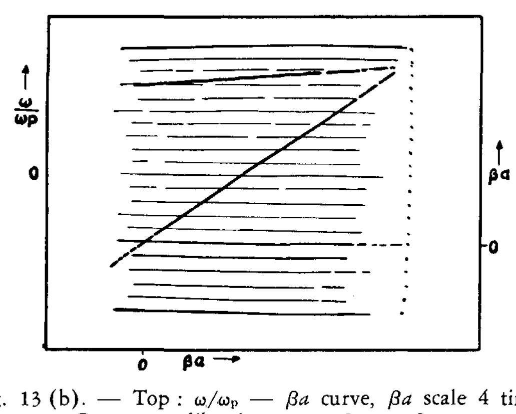

Fig. 13 (b). — Top : ω/ω_p — βa curve, βa scale 4 times
Bottom : calibration curve βa vs. βa.

Case 2. ε_p positive; ν_1, ν_1 and ν_p imaginary.

Let $\nu_1 = i\,\eta_1$, $\nu_2 = i\,\eta_2$ and $\nu_p = i\,\eta_p$, where η_1, η_2 and η_p are real. Using these relations in (5), and writing in non-dimensional form, we get

$$2\,x_1 + [(x_1^2 \frac{m^2 y}{z^2} + \frac{z^2}{m^2 y}) \sinh x_1 + (x_1 \frac{m^2 y}{z^2}) (\frac{x_2 n}{p}) \cosh x_1] \sinh y = [2\,x_1 \cosh x_1 + \frac{x_2 n}{p} \sinh x_1] \cosh y \qquad (13)$$

where $y = \eta_p a$.

The computational technique is the same as in case 1, and therefore no further details are necessary here but to present a representative result in figure 14.

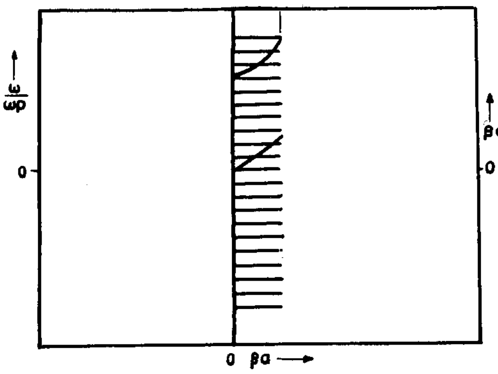

Fig. 14. — Top : ω/ω_p — βa curve. Bottom : calibration curve βa vs. βa.

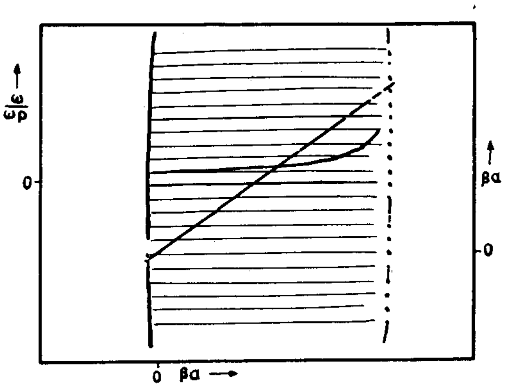

Fig. 15. — Top : ω/ω_p — βa curve, βa scale 5 times. Bottom : calibration curve βa vs. βa.

Case 3. ε_p negative; ν_1, ν_2 and ν_p imaginary.

For this case, the relation (13) is used but the condition that ε_p is negative is imposed appropriately. Figure 15 illustrates a representative curve of ω/ω_p vs. βa.

V. Conclusion

It is evident that a magneto-hydrodynamic problem of this nature, though complex, can be studied by analog computor. In all of the three cases considered, the existence of « forward » slow surface wave is displayed, photographed and studied. However, a « window » study (i.e., a limited band within the wide sweeping range of values considered) would have been extremely useful particularly for $\nu_p a$.

It has been mentioned earlier that the technique used here scans a slow signal by an extremely fast signal. The advantage is elimination of a host of potentiometers or decade boxes and function generators. However, in scanning method difficulties may be experienced with the speed of response of the track and hold circuit in relation to the differences between the sweeping frequencies of the chosen ramp signals.

In this particular problem, the analog computor is used as a calculator rather than simulator. It has solved the physical problem through the medium of mathematics. However, simulation of hydrodynamic or electromagnetic or acoustic problems by analog computor is not quite impossible though it has not been fully explored.

Acknowledgement.

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