# Use of Operational Amplifiers in Precision Current Regulators*

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The design and construction of precision current regulators is simplified by the use of commercially available operational amplifiers. Their method of use is described and an example is given of a regulator used with an electron spectrometer, where the regulation obtained was 0.01%. It is felt that the use of operational amplifiers offers a great convenience to the physics laboratory.

## 1. INTRODUCTION

THE usual method of design for a current regulator is to find a power source that will produce the required current, and then design an error detecting and amplifying circuit with sufficient output to control the power source. This circuit is usually designed in an *ad hoc* manner, and is usually not adaptable to new situations.

The availability of operational amplifiers intended for analog computer service makes possible the design of simply constructed regulator amplifiers that are easily adaptable to many types of power source.

## 2. THE OPERATIONAL AMPLIFIER

The operational amplifier is a low-drift, high-gain dc amplifier that is used as the active element in adders and integrators in electronic analog computers. It gets its name from its ability to perform several linear operations. Some typical operational amplifiers are shown in Fig. 1, with a schematic shown in Fig. 2. The amplifiers are some of several types made by the G. A. Philbrick Researches, Inc.[1]

The basic operational circuit is shown in Fig. 3. The transfer function is given by

$$\frac{E_0}{E_i}=-\frac{Z_f}{Z_i\left(1+\dfrac{Z_f+Z_i}{KZ_i}\right)},\tag{1}$$

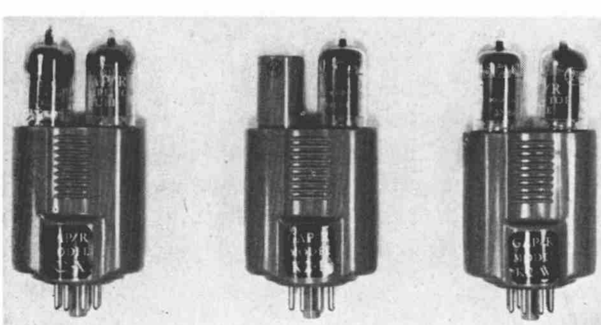

FIG. 1. Typical operational amplifiers.

where $Z_f$ and $Z_i$ are the (Laplace transformed) impedances of the feedback and input networks. If

$$Z_f \ll KZ_i,\tag{2}$$

then

$$E_0=-\frac{Z_f}{Z_i}E_i\tag{3}$$

to sufficient accuracy.

This shows that the circuit may be used as an integrator, since if

$$Z_i=R\tag{4}$$

and

$$Z_f=\frac{1}{CS},\tag{5}$$

then

$$E_0(S)=-\frac{1}{TS}E_i(S);\quad T=RC,\tag{6}$$

or

$$e_0(t)=\frac{-1}{T}\int_0^t e_i(\tau)d\tau-\frac{e_i(0)}{T}.\tag{7}$$

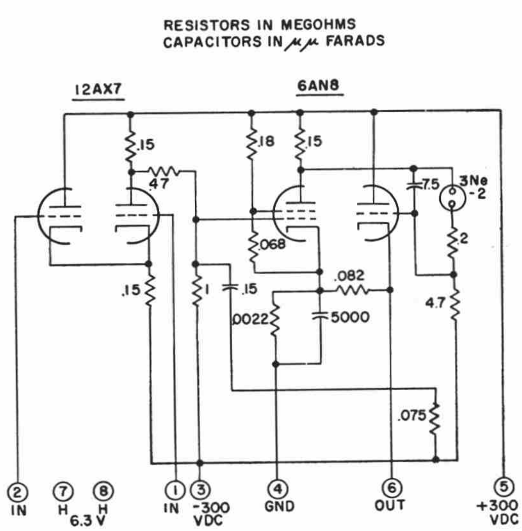


FIG. 2. Schematic of an operational amplifier type K2–X shown in Fig. 1.

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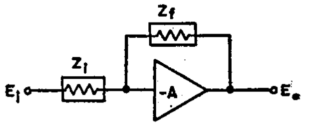


FIG. 3. Basic operational circuit.

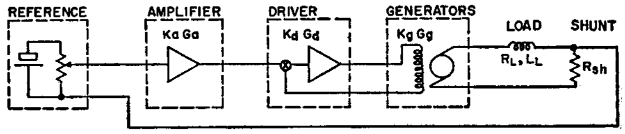


FIG. 4. Regulator block diagram.

### 3. THE REGULATOR

The block diagram of the basic regulator form is shown in Fig. 4. From this the controlled current is given by

$$i_L=\frac{KG}{KG+1}\left(\frac{E_r}{R_{\mathrm{sh}}}\right), \tag{8}$$

where

$$K=K_aK_dK_g, \tag{9}$$

$$G=G_aG_dG_g. \tag{10}$$

The gain ($K$) is a numerical quantity that can easily be adjusted, but the dynamic characteristics of the regulator are determined by the part of the transfer function that depends on the complex frequency $S$. In typical cases these might be

$$G_d=\frac{1}{\tau_dS+1}, \tag{11}$$

where

$$\tau_d=\frac{L_f}{R_fK_{d0}}, \tag{12}$$

and where $L_f$ and $R_f$ refer to the generator field winding inductance and resistance; and $K_{d0}$ is the driver stage minor loop gain (see Appendix I). A typical time constant without minor loop feedback ($K_{d0}=1$) might be of the order of seconds. A minor loop gain of the order of hundreds or thousands is not impractical.

Similarly

$$G_g=\frac{1}{\tau_gS+1}, \tag{13}$$

where

$$\tau_g=\frac{L_a+L_L}{R_a+R_L}. \tag{14}$$

In Eq. (14) the subscript $a$ refers to the generator armature winding and $L$ to the load. These time constants may also be of the order of seconds.

The types of amplifier transfer functions to be considered are

$$G_{a1}=1, \tag{15}$$

$$G_{a2}=\frac{1+aS}{S}. \tag{16}$$

In the second case the amplifier is said to be an augmented integrator, since it adds a linear term to the integrated output.

The advantage of the second system can be found by considering the response to a step input, and using the final value theorem to obtain the asymptote of the response. When this is done for the ordinary amplifier [Eq. (15)],

$$i_L(t\to\infty)=\lim_{S\to0}\frac{K(E_i/R_{\mathrm{sh}})}{K+(\tau_dS+1)(\tau_gS+1)}=\frac{K}{K+1}\left(\frac{E_i}{R_{\mathrm{sh}}}\right); \tag{17}$$

whereas in the second case [Eq. (16)],

$$i_L(t\to\infty)=\lim_{S\to0}\frac{K(1+aS)(E_i/R_{\mathrm{sh}})}{K(1+aS)+S(\tau_dS+1)(\tau_gS+1)}=\left(\frac{E_i}{R_{\mathrm{sh}}}\right); \tag{18}$$

so that in the circuit that uses the integrator there is no steady-state error, and the asymptotic output is independent of the regulator gain. In a similar manner it can be shown that there is no error due to drifts in components placed after the integrator.

These advantages have compensating disadvantages, which are two-fold. First, the additional power of $S$ in the transfer function puts more stringent limits on the maximum gain for stability; and second, the very poor

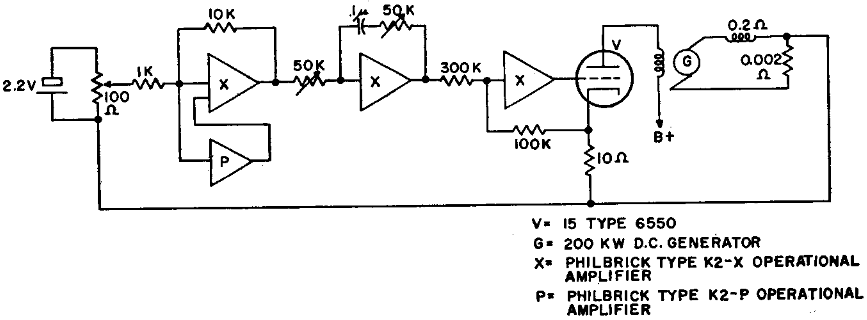


FIG. 5. A regulator for an electron spectrometer.

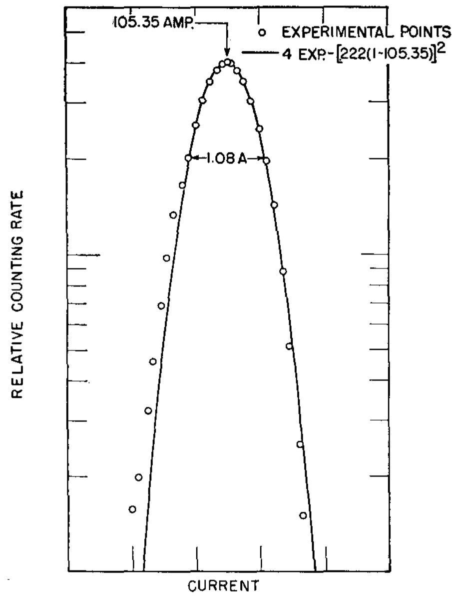


FIG. 6. Conversion line from the 1.06-Mev transition in $Bi^{207}$.

high-frequency response of the integrator makes the regulation against high-frequency noise somewhat poorer. The use of the augmented integrator, i.e., with

$$a \neq 0$$

in Eq. (16), reduces both effects, since it acts to damp the system and also has a constant minimum gain at high frequencies.

## 4. A CURRENT REGULATOR FOR AN ELECTRON SPECTROMETER

An example of a regulator designed to use operational amplifiers is shown in the schematic of Fig. 5. This regulator was designed around the 200-kw marine generator that provided the power source. The load is the coil assembly of an intermediate image pair spectrometer. The driver stage uses 15 type 6550 tubes to supply the 3 amp needed in the generator field winding. The input stage was chopper stabilized for low drift (Appendix II).

The variation in output was found to be less than 0.01 amp or 0.01% (whichever was larger) over the range from 30 to 1000 amp; with a drift rate of about 0.01% per hour due to drifts in the reference battery.

As an example of the use of this regulator, Fig. 6 shows a spectrum taken with the spectrometer on the conversion line of the 1.06-Mev transition in $Bi^{207}$. The number of points that can be taken allows a precise evaluation of line position, since by finding the center of symmetry of the line a position is obtained with an error of only 3% of the line width. The spectrometer was used to make several measurements of the ratio of currents corresponding to the conversion electrons from the 1.06- and 0.57-Mev transitions, with the result that

$$\frac{I_{1.06}}{I_{0.57}} = 1.64118 \pm 0.00039.$$

The results of the best recent measurements on these lines,[2,3] taken with spectrometers of somewhat better resolution, gave a momentum ratio of

$$\frac{H\rho_{1.06}}{H\rho_{0.57}} = 1.64074 \pm 0.00042$$

and the excellent agreement testifies both to the linearity of the spectrometer and the precision possible with a highly regulated current supply.

## APPENDIX I. MODIFICATION OF TIME CONSTANTS BY FEEDBACK

An example of two circuits that have approximately the same gain but in one of which minor loop feedback is used to reduce the time constant is shown in Fig. 7. In Fig. 7(a)

$$\frac{i}{E_i} = \frac{1}{R_L(\tau_a S + 1)}, \tag{A-1}$$

where

$$\tau_a = \frac{L_L}{R_L}; \tag{A-2}$$

whereas in Fig. 7(b),

$$\frac{i}{E_i} = \frac{1}{R_L\left(1 + \frac{1}{K} + \frac{R_{sh} + r_p + R_L}{\mu K R_{sh}}\right)(\tau_b S + 1)}, \tag{A-3}$$

where

$$\tau_b = \frac{L}{R_L + r_p + (1 + \mu + K\mu) R_{sh}}, \tag{A-4}$$

and where the open loop gain is approximately

$$K_0 \approx K\mu \frac{R_{sh}}{R_L}. \tag{A-5}$$

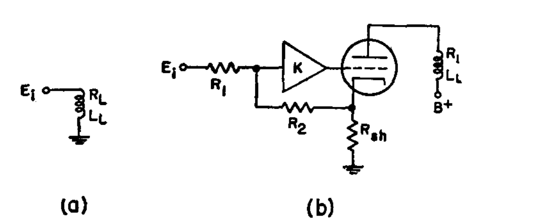


FIG. 7. Inductive load without (a) and with (b) minor loop feedback. In (b) $R_2 \gg R_{sh}$ and $R_1 : R_2 = R_L : R_{sh}$.

[2] D. A. Alburger, Phys. Rev. **92**, 1257 (1953).
[3] G. Bäckström, Arkiv Fysik **10**, 393 (1956).

## APPENDIX II. THE CHOPPER STABILIZED AMPLIFIER

In the circuit shown in Fig. 8 a moderate gain, very low drift amplifier is used to reduce the effects of grid drifts. This amplifier is usually of the modulator-carrier amplifier-demodulator type, with the modulation done with a mechanical interrupter (chopper). It is used with an operational amplifier that has differential inputs. In Fig. 8 these are marked + and −. If the effect of grid drift is represented by an additional error voltage ($e$) in series with one input, the output is given by

$$E_0=\frac{-K'E_i}{1+\dfrac{1+K'}{K(A+1)}}-\frac{e}{\dfrac{1}{K}+\dfrac{A+1}{K'+1}}, \qquad \text{(A-6)}$$

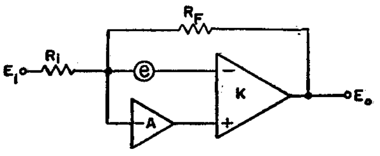


FIG. 8. Chopper stabilized operational amplifier.

or

$$E_0 \simeq -K'\left(E_i+\frac{e}{A+1}\right), \qquad \text{(A-7)}$$

where

$$K'=R_f/R_i. \qquad \text{(A-8)}$$

Thus the effect of the stabilizing amplifier is to reduce the effects of drifts by the amount of the amplifier gain.